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### Linear Algebra Websites and Email Address

- [math.mit.edu/linearalgebra](http://math.mit.edu/linearalgebra) Dedicated to readers and teachers working with this book  
[ocw.mit.edu](http://ocw.mit.edu) MIT's OpenCourseWare site including video lectures in 18.06 and 18.085-6  
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